Introduction to Data-Flow and Control-Flow Analyses

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Outline of the Lecture

- Introduction
- Examples of data-flow analyses
- Fundamentals of control-flow analysis
- Interprocedural data-flow analysis
- Points-to analysis

Data-flow analysis

- These are techniques that derive information about the flow of data along program execution paths.
- An *execution path* (or *path*) from point p_1 to point p_n is a sequence of points $p_1, p_2, ..., p_n$ in the program that occur one after another.
- In general, there are infinite number of paths through a program and there is no bound on the length of a path.

Data-flow analysis

- Program analyses summarize all possible program states that can occur at a point in the program with a finite set of facts.
- No analysis is necessarily a perfect representation of the state.
- DFAs collect information necessary to perform program optimizations.

Uses of Data-flow Analysis

- Program debugging
  - Which are the definitions (of variables) that *may* reach a program point? These are the *reaching definitions*.
- Program optimizations
  - Constant folding
  - Copy propagation
  - Common sub-expression elimination etc.

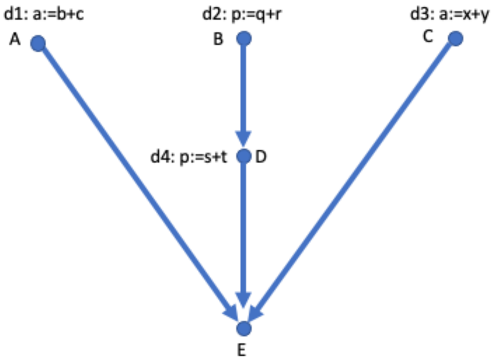
Data-Flow Analysis Schema(1)

- A *data-flow value* for a program point represents an abstraction of the set of all possible program states that can be observed for that point.
- The set of all possible data-flow values is the *domain* for the application under consideration.
  - Example: for the *reaching definitions* problem, the domain of data-flow values is the set of all subsets of of definitions in the program.
  - A particular data-flow value is a set of definitions.
- $IN[B]$ and $OUT[B]$: data-flow values *before* and *after* each basic block B.
- The *data-flow problem* is to find a solution to a set of constraints on $IN[B]$ and $OUT[B]$, for all basic blocks B.

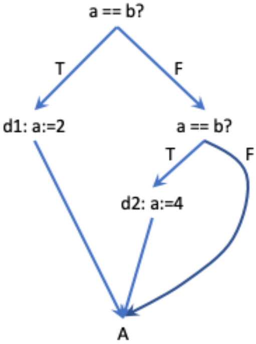
Data-Flow Analysis Schema (2)

- Two kinds of constraints
  - Those based on the semantics of statements (*transfer functions*).
  - Those based on flow of control.
- A DFA schema consists of
  - A control-flow graph.
  - A direction of data-flow (forward or backward).
  - A set of data-flow values.
  - A confluence operator (usually set union or intersection).
  - Transfer functions for each block.
- We always compute *safe* estimates of data-flow values.
- A decision or estimate is *safe* or *conservative*, if it never leads to a change in what the program computes (before and after using the estimated value).
- These safe values may be either subsets or supersets of actual values, based on the application.

The Reaching Definitions Problem(1)



d1, d3, and d4 *reach* E.
d2 reaches only D. d4 *kills* d2.



Both d1 and d2 *reach* A. This is safe. It is safe to assume that a definition reaches a point even if it does not.

The Reaching Definitions Problem(2)

- We *kill* a definition of a variable *a*, if between two points along the path, there is an assignment to *a*.
- A definition *d* *reaches* a point *p*, if there is a path from the point immediately following *d* to *p*, such that *d* is not *killed* along that path.
- We compute supersets of definitions as *safe* values.
- It is safe to assume that a definition reaches a point, even if it does not.
- In the following example, we assume that both `a=2` and `a=4` reach the point after the complete if-then-else statement, even though the statement `a=4` is not reached by control flow:

```
if (a==b) a=2; else if (a==b) a=4;
```

The Reaching Definitions Problem (3)

- The data-flow equations (constraints)

$$
\begin{aligned}
IN[B] &= \bigcup_{P \text{ is a predecessor of } B} OUT[P] \\
OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B]) \\
IN[B] &= \phi, \textit{ for all } B \ (\textit{initialization only})
\end{aligned}
$$

- If some definitions reach B_1 (entry), then $IN[B_1]$ is initialized to that set.
- Forward flow DFA problem (since $OUT[B]$ is expressed in terms of $IN[B]$), confluence operator is $\bigcup$.
  - Direction of flow does not imply traversing the basic blocks in a particular order.
  - The final result does not depend on the order of traversal of the basic blocks.

The Reaching Definitions Problem (4)

- $GEN[B]$ = set of all definitions inside B that are "visible" immediately after the block - *downwards exposed* definitions
  - If a variable x has two or more defintions in a basic block, then only the last definition of x is downwards exposed; all others are not visible outside the block
- $KILL[B]$ = union of the definitions in all the basic blocks of the flow graph, that are killed by individual statements in B
  - If a variable x has a definition d_i in a basic block, then d_i kills all the definitions of the variable x in the program, except d_i

Reaching Definitions Analysis: GEN and KILL

In other blocks:

d5: b = a+4
d6: f = e+c
d7: e = b+d
d8: d = a+b
d9: a = c+f
d10: c = e+a

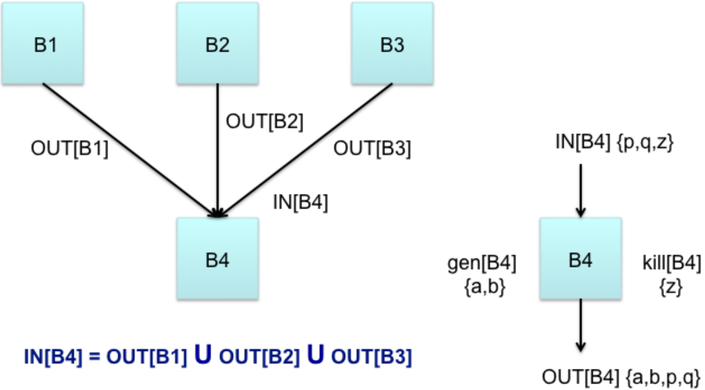
d1: a = f + 1
d2: b = a + 7
d3: c = b + d
d4: a = d + c

B

Set of all definitions = {d1,d2,d3,d4,d5,d6,d7,d8,d9,10}

GEN[B] = {d2,d3,d4}
KILL[B] = {d4,d9,d5,d10,d1}

Reaching Definitions Analysis: DF Equations



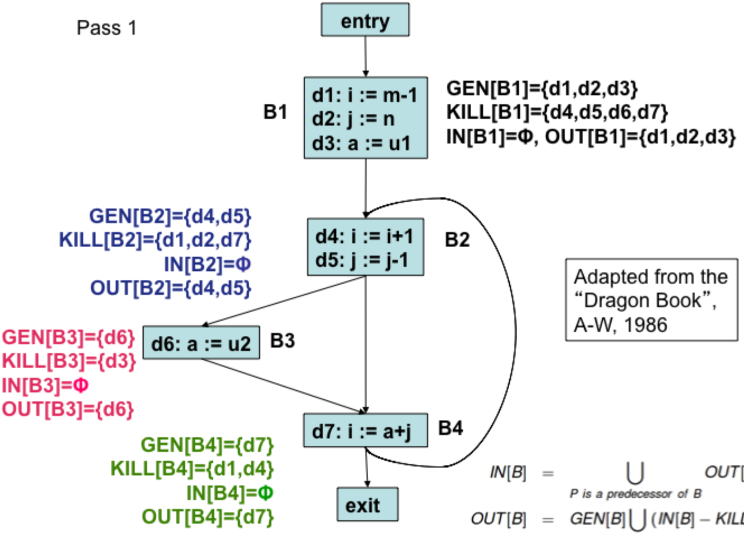
IN[B4] = OUT[B1] U OUT[B2] U OUT[B3]

$$
\begin{aligned}
IN[B] &= \bigcup_{P \text{ is a predecessor of } B} OUT[P] \\
OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B])
\end{aligned}
$$

OUT[B4] = gen[B4] U (IN[B4] – kill[B4])

Reaching Definitions Analysis: An Example - Pass 1

Pass 1



Adapted from the "Dragon Book", A-W, 1986

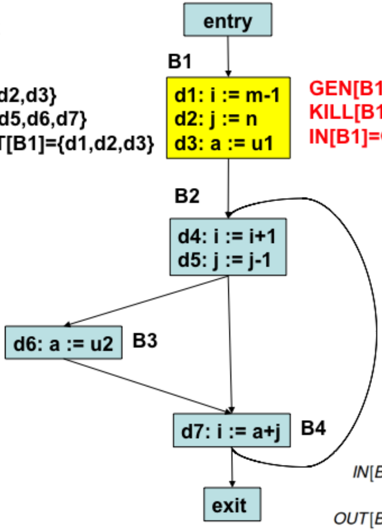
$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Reaching Definitions Analysis: An Example - Pass 2.1

Pass 2

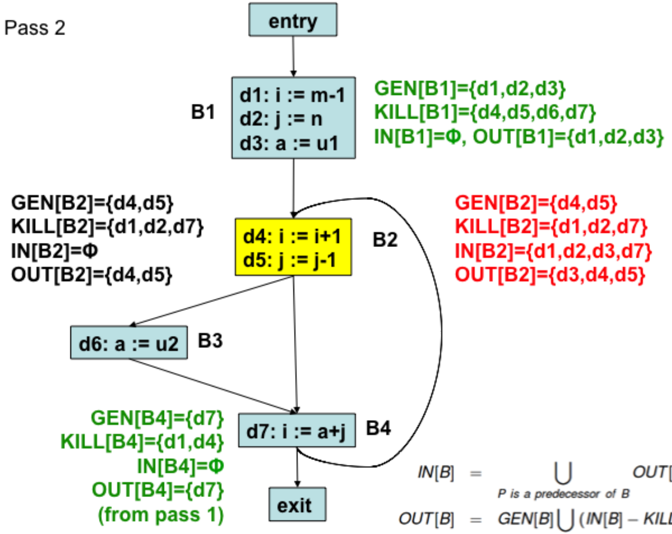
GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

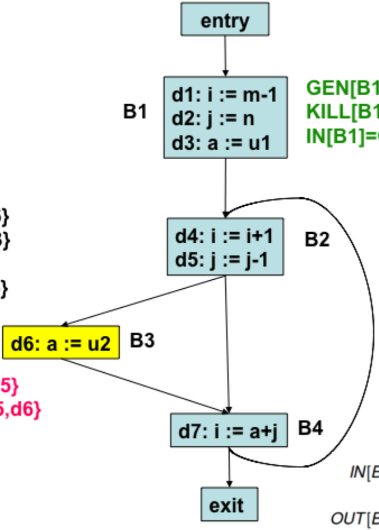


$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Reaching Definitions Analysis: An Example - Pass 2.3

Pass 2



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]={d1,d2,d3,d7}
OUT[B2]={d3,d4,d5}

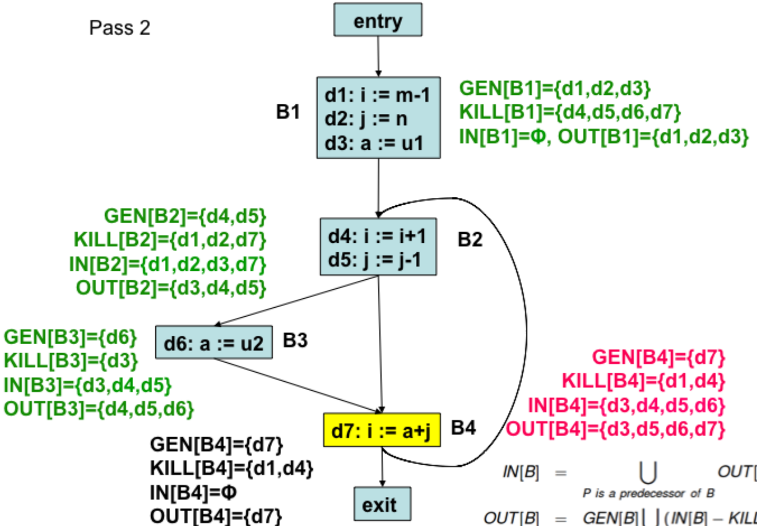
GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]=Φ
OUT[B3]={d6}

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]={d3,d4,d5}
OUT[B3]={d4,d5,d6}

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Pass 2

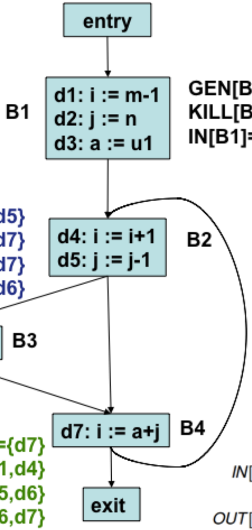


$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

Reaching Definitions Analysis: An Example - Final

Final



GEN[B1]={d1,d2,d3}
KILL[B1]={d4,d5,d6,d7}
IN[B1]=Φ, OUT[B1]={d1,d2,d3}

GEN[B2]={d4,d5}
KILL[B2]={d1,d2,d7}
IN[B2]={d1,d2,d3,d5,d6,d7}
OUT[B2]={d3,d4,d5,d6}

GEN[B3]={d6}
KILL[B3]={d3}
IN[B3]={d3,d4,d5,d6}
OUT[B3]={d4,d5,d6}

GEN[B4]={d7}
KILL[B4]={d1,d4}
IN[B4]={d3,d4,d5,d6}
OUT[B4]={d3,d5,d6,d7}

Adapted from the "Dragon Book", A-W, 1986

$$IN[B] = \bigcup_{P \text{ is a predecessor of } B} OUT[P]$$

$$OUT[B] = GEN[B] \bigcup (IN[B] - KILL[B])$$

An Iterative Algorithm for Computing Reaching Def.

for each block B do { $IN[B] = \phi$; $OUT[B] = GEN[B]$; }
$change = true$;
while $change$ do { $change = false$;
for each block B do {

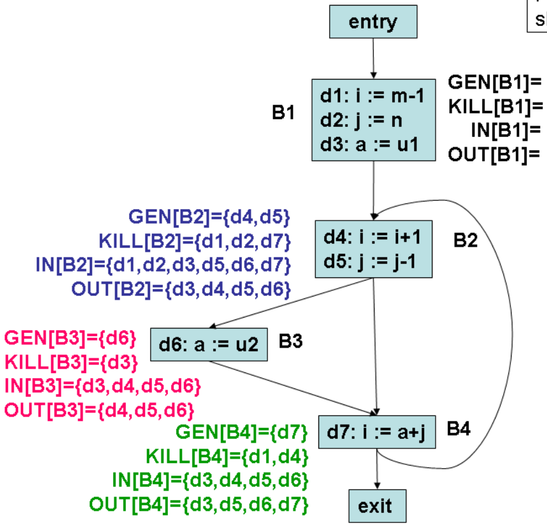
$$
\begin{aligned}
IN[B] &= \bigcup_{P \text{ a predecessor of } B} OUT[P]; \\
oldout &= OUT[B]; \\
OUT[B] &= GEN[B] \bigcup (IN[B] - KILL[B]);
\end{aligned}
$$

if ($OUT[B] \neq oldout$) $change = true$;
}
}

- *GEN*, *KILL*, *IN*, and *OUT* are all represented as bit vectors with one bit for each definition in the flow graph

Reaching Definitions: Bit Vector Representation

Final dataflow value sets shown in bit vector format



	d1	d2	d3	d4	d5	d6	d7
GEN[B1]=	1	1	1	0	0	0	0
KILL[B1]=	0	0	0	1	1	1	1
IN[B1]=	0	0	0	0	0	0	0
OUT[B1]=	1	1	1	0	0	0	0

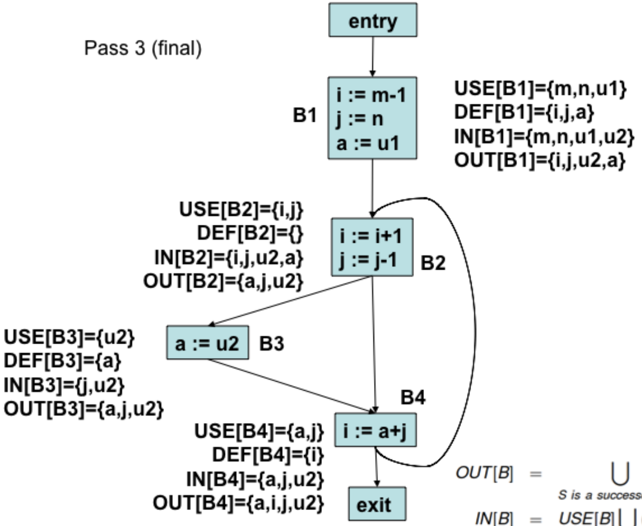
Adapted from the “Dragon Book”, A-W, 1986

Live Variable Analysis (1)

- The variable x is *live* at the point p, if the value of x at p could be used along some path in the flow graph, starting at p; otherwise, x is *dead* at p.
- Sets of variables constitute the domain of data-flow values.
- Backward flow problem, with confluence operator $\bigcup$.
- $IN[B]$ is the set of variables live at the beginning of B.
- $OUT[B]$ is the set of variables live just after B.
- $DEF[B]$ is the set of variables definitely assigned values in B, prior to any use of that variable in B.
- $USE[B]$ is the set of variables whose values may be used in B prior to any definition of the variable.

$$
\begin{aligned}
OUT[B] &= \bigcup_{S \text{ is a successor of } B} IN[S] \\
IN[B] &= USE[B] \bigcup (OUT[B] - DEF[B]) \\
IN[B] &= \phi, \textit{ for all } B \textit{ (initialization only)}
\end{aligned}
$$

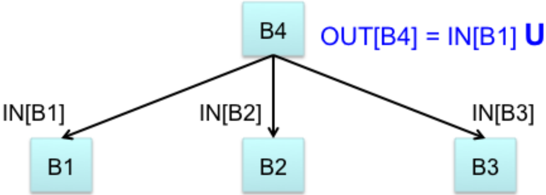
Live Variable Analysis: An Example



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

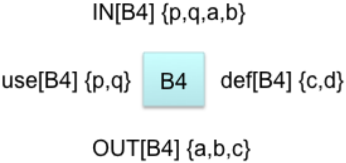
$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Live Variable Analysis (2)



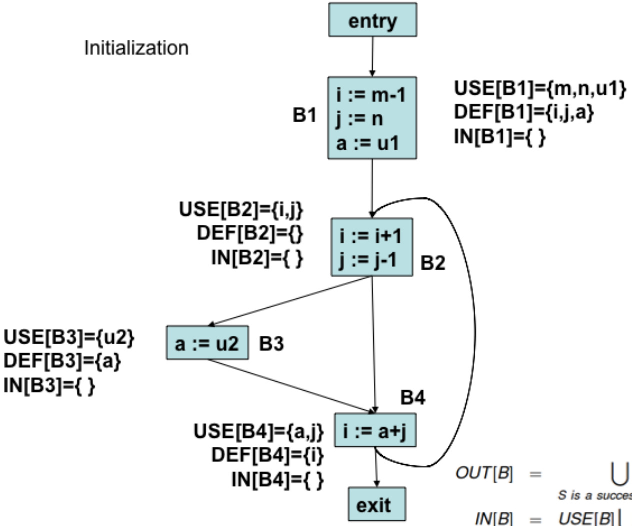
OUT[B4] = IN[B1] **U** IN[B2] **U** IN[B3]

$$
\begin{aligned}
OUT[B] &= \bigcup_{S \text{ is a successor of } B} IN[S] \\
IN[B] &= USE[B] \bigcup (OUT[B] - DEF[B]) \\
IN[B] &= \phi, \text{for all } B \text{ (initialization only)}
\end{aligned}
$$



IN[B4] = use[B4] **U** (OUT[B4] – def[B4])

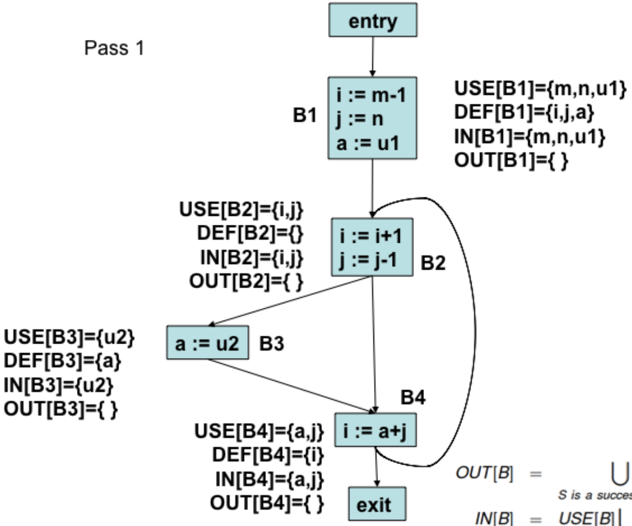
Live Variable Analysis: An Example - Init



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

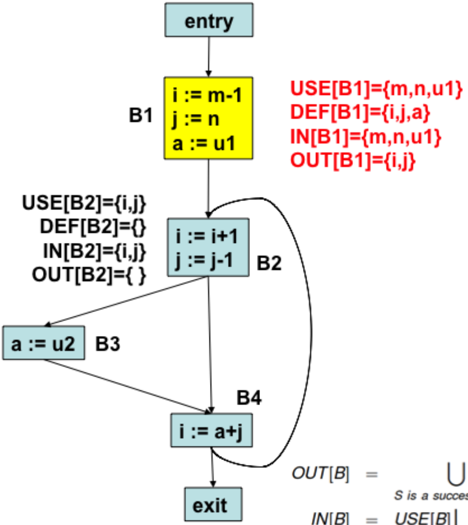
Live Variable Analysis: An Example - Pass 1



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

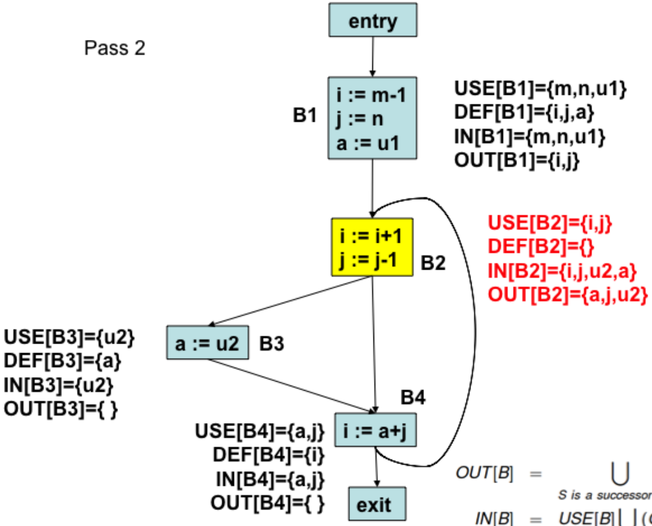
Pass 2



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

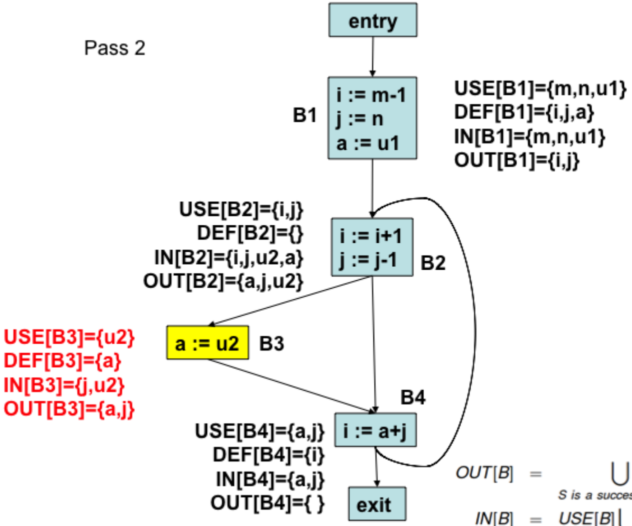
Live Variable Analysis: An Example - Pass 2.2



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

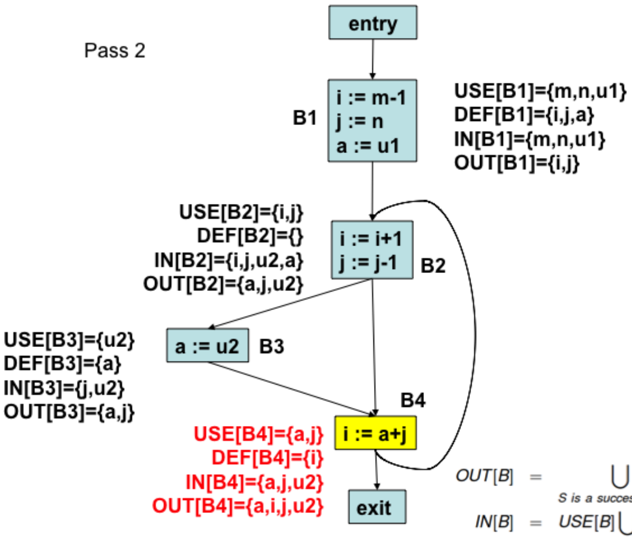
Pass 2



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

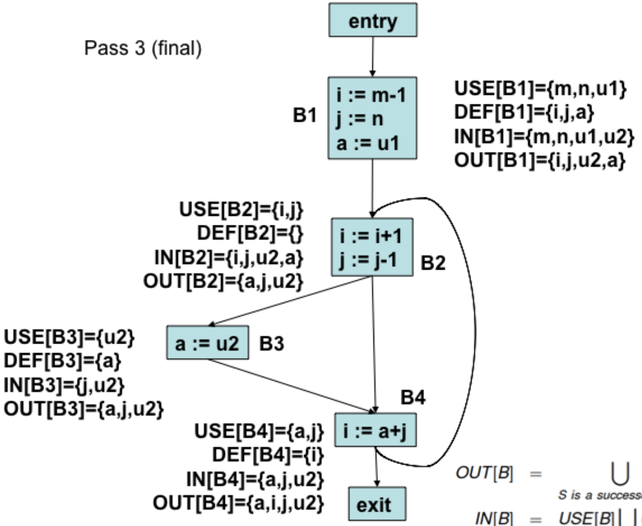
$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Live Variable Analysis: An Example - Pass 2.4



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$



$$OUT[B] = \bigcup_{S \text{ is a successor of } B} IN[S]$$

$$IN[B] = USE[B] \bigcup (OUT[B] - DEF[B])$$

Control-Flow Analysis

Introduction

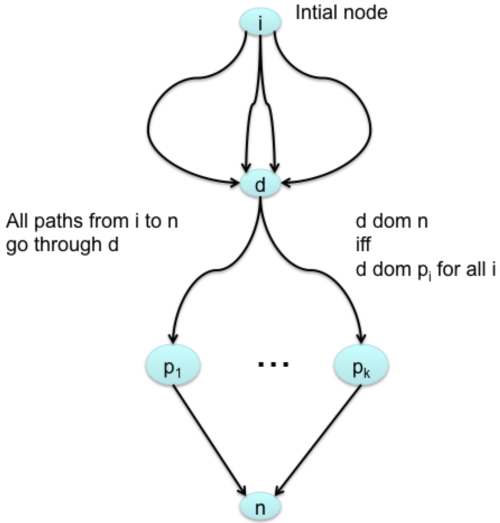
Control-flow analysis (CFA) helps us to understand the structure of control-flow graphs (CFG).

- To determine the loop structure of CFGs.
- To compute dominators - useful for code motion.
- To compute dominance frontiers - useful for the construction of the static single assignment form (SSA).
- To compute control dependence - needed in parallelization.

Dominators

- We say that a node d in a flow graph *dominates* node n, written d *dom* n, if every path from the initial node of the flow graph to n goes through d.
- Initial node is the root, and each node dominates only its descendents in the dominator tree (including itself).
- Principle of the dominator algorithm
  - If $p_1, p_2, ..., p_k$, are all the predecessors of n, and $d \neq n$, then d *dom* n, iff d *dom* p_i for each i.

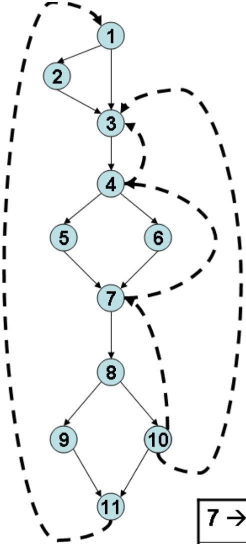
Dominator Algorithm Principle



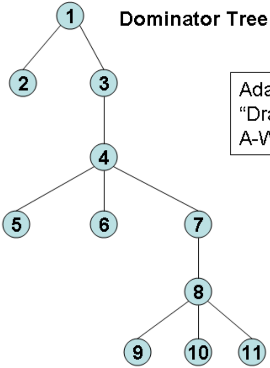
Dominators and Natural Loops

- Edges whose heads dominate their tails are called *back edges* ($a \rightarrow b:\ b = head,\ a = tail$)
- Given a back edge $n \rightarrow d$
  - The *natural loop* of the edge is d plus the set of nodes that can reach n without going through d
  - d is the header of the loop
    - A single entry point to the loop that dominates all nodes in the loop
    - At least one path back to the header exists (so that the loop can be iterated)

Dominators, Back Edges, and Natural Loops(1)



Flow Graph



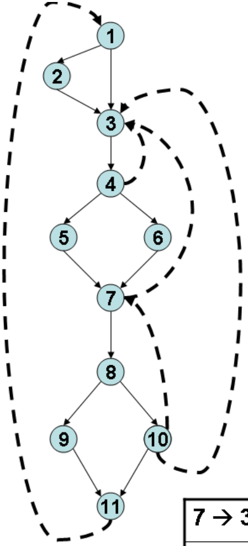
Dominator Tree

Adapted from the "Dragon Book", A-W, 1986

Back edges and their natural loops

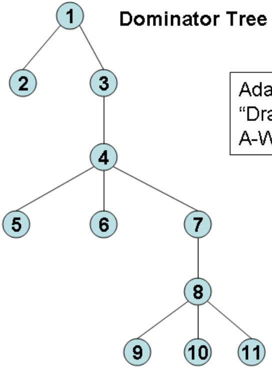
7 → 4	10→7	4→3	10→3	11→1
{4,5,6,7,8, 10}	{7,8,10}	{3,4,5,6,7, 8,10}	{3,4,5,6,7, 8,10}	{1,2,3,4,5, 6,7,8,9,10,11}

Dominators, Back Edges, and Natural Loops(2)



Flow Graph

Dominator Tree



Adapted from the "Dragon Book", A-W, 1986

Back edges and their natural loops

7 → 3	10→7	4→3	10→3	11→1
{3,4,5,6,7,8, 10}	{7,8,10}	{3,4}	{3,4,5,6,7, 8,10}	{1,2,3,4,5, 6,7,8,9,10,11}

Interprocedural Data-Flow Analysis

Introduction

Intraprocedural DFA

- Performed on one procedure at a time.
- Assumes that a procedure invocation may alter all the "visible" variables.
- Imprecise, conservative, but simple.

Interprocedural DFA

- Operates across an entire program and is more complex.
- Makes information flow from caller to callee and vice-versa.
- Procedure inlining is a simple method to enable such information flow.
  - Applicable only if target of a call is known.
  - Increases memory footprint.
- Applications
  - Converting virtual method calls to static method calls.
  - Checking array bounds.
  - Better constant propagation.

Interprocedural DFA

Context-Insensitive IDFA

- Treat each call and return as goto operations.
- Create a super control flow graph .
- Apply standard analysis on the super CFG.
- Simple, but imprecise.
  - A function is analyzed as a common entity for all its calls.
  - Only input-output behaviour of a function is abstracted out.

Context-Sensitive IDFA

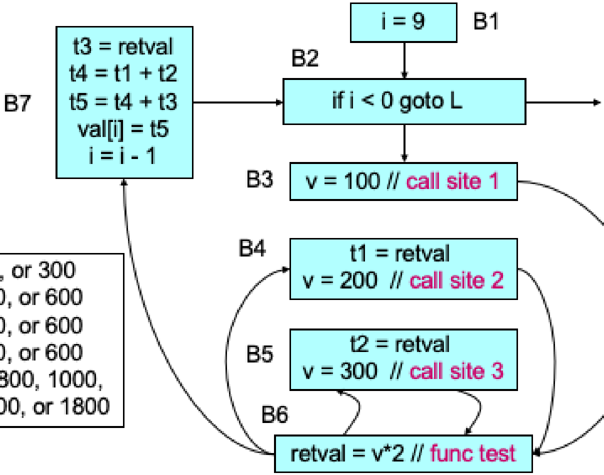
- Calling context (call site) determines the effect of the call.
- In the example (next slide), function test is invoked with a constant in each of the call sites, but the value of the constant is context-dependent.

IDFA - Example Program

```
i = 9;
while (i >= 0) {
      t1 = test(100); // call site 1
      t2 = test(200); // call site 2
      t3 = test(300); // call site 3
      val[i--] = t1 + t2 + t3;
}
int test (int v) {
    return (v*2);
}
```

Context-insensitive IDFA: function test is deemed to return 200, 400, or 600 for any of the three calls.
Context-sensitive IDFA: with interprocedural constant propagation, t1, t2, and t3 will get precisely 200, 400, and 600 respectively.

Super CFG and Context-Insensitive IDFA



v (B6): 100, 200, or 300
t1 (B4): 200, 400, or 600
t2 (B5): 200, 400, or 600
t3 (B7): 200, 400, or 600
val[i] (B7): 600, 800, 1000, 1200, 1400, 1600, or 1800

Cloning and Context-Sensitive IDFA

Simple, context-insensitive analysis is enough on the cloned call graph

```
i = 9;
while (i >= 0) {
    t1 = f1 (100); // call site c1
    t2 = f2 (200); // call site c2
    t3 = f3 (300); // call site c3
    val[i--] = t1 + t2 + t3;
}
int f1 (int v) {
    return test1 (v); // call site c4.1
}
int test1 (int v) {
  return (v*2);
}
```

```
int f2 (int v) {
    return test2 (v); // call site c4.2
}
int test2 (int v) {
   return (v*2);
}
int f3 (int v) {
    return test3 (v); // call site c4.3
}
int test3 (int v) {
   return (v*2);
}
```

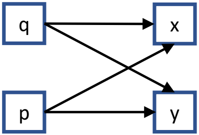
Recursive programs cannot be handled

Points-To Analysis

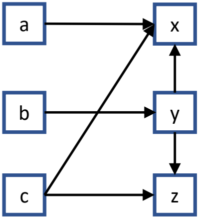
Introduction

Points-to analysis discovers what each pointer variable and heap reference points to.

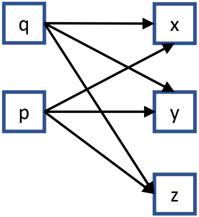
```
q := &x
q := &y
p := q
```




```
a := &x
b := &y
if p then
    y := &z
else
    y := &x
fi
c := *y
```

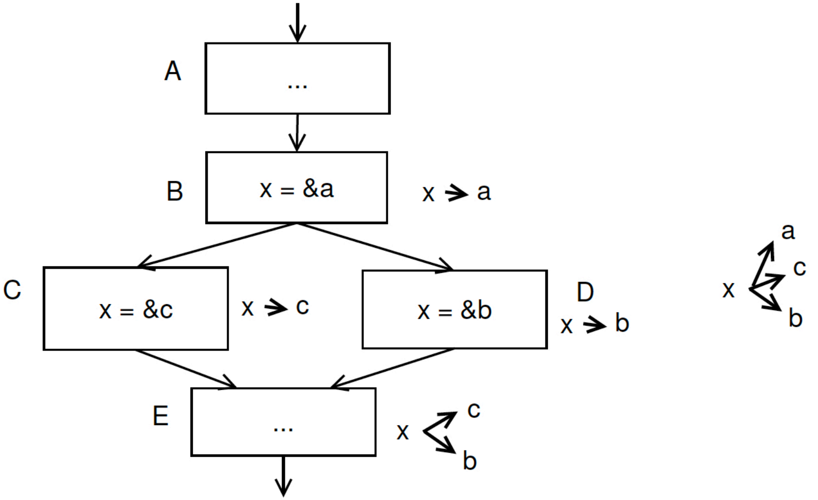



```
q := &x
q := &y
p := q
q := &z
```



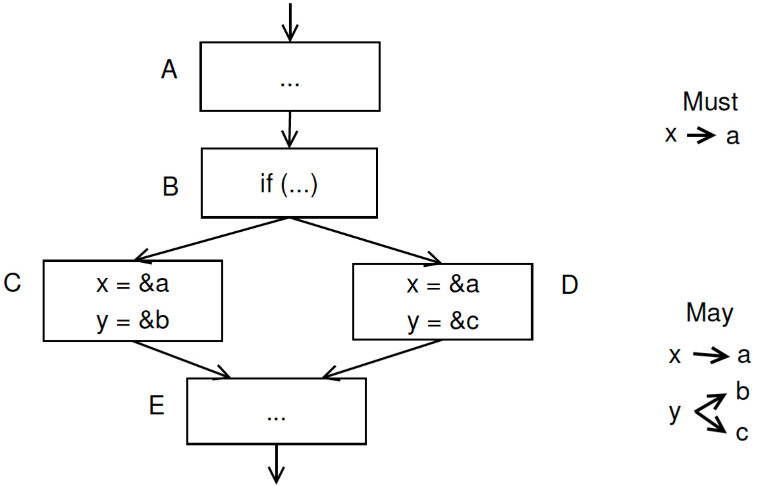
These are flow-insensitive analyses.

Flow-Insensitive vs. Flow-Sensitive Analysis



courtesy: Subhajit Roy

May vs. Must Analysis



courtesy: Subhajit Roy